QUESTIONS

RELATING TO

SCHEMES

FOR GRANTING

REVERSIONARY ANNUITIES.

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SCHEMES

FOR CALCULATING

REVERSIONARY ANNUITIES.

QUESTIONS

RELATING TO

SCHEMES

FOR GRANTING

REVERSIONARY ANNUITIES,

TOGETHER WITH SOME

OBSERVATIONS

ON

ANNUITY SCHEMES.

Particularly that Eſtabliſhed by Act of Parliament, for Raiſing and Eſtabliſhing a Fund, for a Proviſion for the WIDOWS and CHILDREN of the Miniſters of the CHURCH of SCOTLAND. On which was grounded a Plan lately propoſed to be Eſtabliſhed by Act of Parliament, for Raiſing and Eſtabliſhing a Fund, for a Proviſion for the WIDOWS and CHILDREN of the Clergy of the CHURCH of IRELAND.
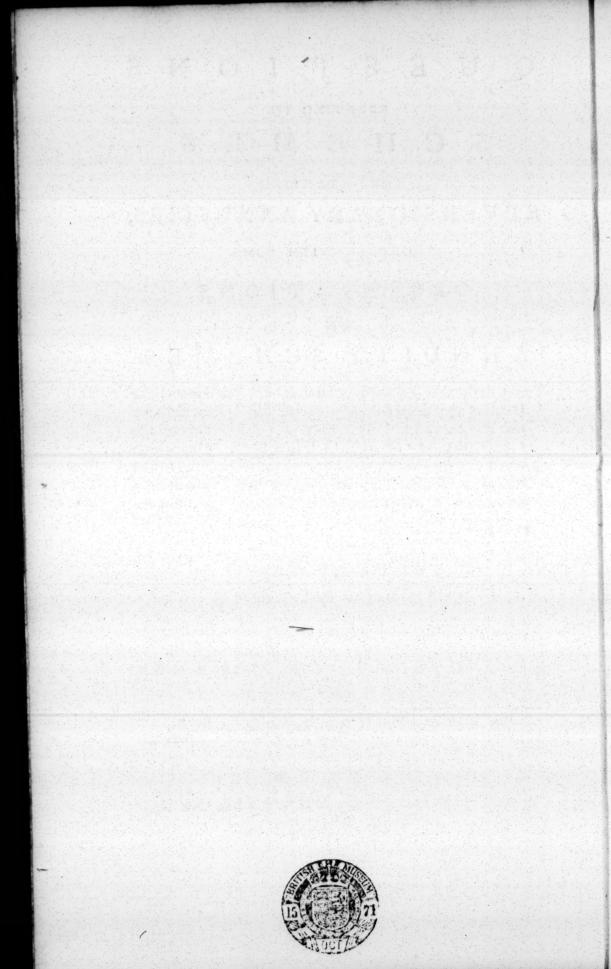
EXTRACTED FROM

OBSERVATIONS

ON

REVERSIONARY PAYMENTS, &c.

By RICHARD PRICE, D. D. F. R. S.

QUESTIONS

RELATING TO

SCHEMES

FOR GRANTING

REVERSIONARY ANNUITIES.

CHAP I.

QUESTION I.

" A Set of married men enter into a fociety for
" fecuring annuities to their widows.   What fum
" of money in a fingle *prefent* payment, ought
" every member to contribute, in order to entitle
" his widow to an annuity of 30*l*. *per ann.* for her
" life, eftimating intereft at 4 *per cent ?*"

ANSWER.

It is evident that the value of fuch an expecta-
tion is different, according to the different ages of

the

the purchafers, and the proportion of the age of the wife to that of the hufband. Let us then fuppofe, that every perfon in fuch a fociety is of the fame age with his wife, and that one with another all the members when they enter may be reckoned 40 years of age, as many entering above this age as below it. It has been demonftrated by Mr. *De Moivre* and Mr. *Simpfon*, that " the value of " an annuity on the *joint continuance* of any two " lives, fubtracted from the value of an annuity " on the life in expectation," gives the true prefent value of an annuity on what may happen to remain of the latter of the two lives after the other.

In the prefent cafe, the value of an annuity to be enjoyed during the *joint continuance* of two lives, each (*a*) 40, (*b*) is 9.826, according to the probabilities

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(*a*) See Table II.

(*b*) The value of *joint* lives and reverfions, as deduced from the *Breflaw* obfervations, are not given in any Part of this work, from Mr. *De Moivre*'s rules in his treatife on annuities on lives. For thefe rules are approximations, which give refults fo far from the truth, as to be, not only ufelefs, but dangerous.

Mr. *De Moivre* has calculated the values of *fingle* lives, on the fuppofition of an *equal decrement of life* thro' all its ftages till the age of 86, which he confidered as the utmoft probable extent of life. Thus; let there be 56 perfons alive at 30 years of age. It is fuppofed that one will die every year till, in 56 years, they will be all dead. The fame will happen to 46 at 40, in 46 years, To 36 at 50, in 36 years, and fo on for

bilities of life in the Table of Obfervations formed by Dr. *Halley*, from the bills of mortality of *Bref-law* in *Silefia*. The value of a fingle life 40 years of age, as given by M. *De Moivre*, agreeably to the fame Table, is 1 3.20 (*c*); and the former fub-tracted from the latter, leaves 3.37, or the true number

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for all other ages. The number of years which a given life wants of 86, he calls the *complement* of that life. Fifty-fix, therefore, is the *complement* of 30; 46 of 40, and 36 of 50.

This hypothefis eafes very much the labour of calculating the values of lives; and at moft ages between thirty and fe-venty, or feventy-five, it is fo conformable to Dr. *Halley*'s ta-ble of obfervations, that I fhall not in thefe queftions think it neceffary to diftinguifh between the values of fingle lives as deduced from this Table, and the fame values deduced from the hypothefis.

Two tables which I have formed from the bills of morta-lity at *Northampton* and *Norwich*, anfwer more nearly to Mr. *De Moivre*'s hypothefis than even Dr. *Halley*'s table; and the difference between the values of *fingle* and *joint* lives by the *hypothefis*, and the fame values computed ftrictly from the tables, is generally lefs in thefe tables than in Dr. *Halley*'s. When, therefore, the values of *fingle* and *joint* lives are mentioned, as given agreeably to Dr. *Halley*'s table; it muft be underftood, that they are taken from tables I and II. and given in ftrict agreement only to the *hypothefis*; and that for this reafon, they are in reality ftill more con-formable to the *Northampton* and *Norwich* tables.

The inhabitants of *London*, as is well known, not living fo long as the reft of mankind, the values of *fingle* aud *joint* lives there, are confiderably lefs than in thofe juft mentioned.

(*c*) See Table I.

number of the years purchafe which ought to be
paid for any given annuity, to be enjoyed by a
perfon of 40 years of age, *provided* he furvives
another perfon of the fame age, intereft being
reckoned at four *per cent. per annum.* The annuity,
therefore propofed in this queftion being 30*l.*
the prefent value of it is 30 multiplied by 3.37, or
101*l.* 2*s.*

By calculating from Mr. *Simpfon*'s Tables form-
ed from the bills of mortality of *London,* this value
comes out 102*l.*

The difference in the value of the reverfion will
be inconfiderable, whether the common age is taken
a few years more or lefs than 40. Thus, married
men of 30 ought not, according to Dr. *Halley*'s
Table, to give two fifths of a year's purchafe more,
for any given reverfionary annuity for their wives,
than married men of 50, provided they are of the
fame ages with their wives; and one quarter more,
according to Mr. *Simpfon*'s Table. If the wives
are younger (as is generally the cafe) there will
indeed be a confiderable difference; for the value
now determined would be 120*l.* according to the
Breflaw obfervations, fuppofing the two lives to be
40 or 33, or that wives are one with another 7
years younger than their hufbands; and 118*l.* 10*s.*
according to the *London* obfervations.

Question II.

" Suppofing fuch a fociety as that defcribed in
" the preceding queftion, to be limited to a cer-
tain

" tain number of members, and conftantly kept
" up to that number, by the admiffion of new
" members as old ones are loft, in confequence
" of their own deaths, and the deaths of their
" wives: What is the number of annuitants
" which, in fome time after its eftablifhment, will
" come to be conftantly upon it?"

ANSWER.

Since every marriage produces either a widow
or widower; and fince all marriages taken together
would produce as many widows as widowers, were
every man and his wife of the fame age, and the
chance equal which fhall die firft; it is evident,
that the number of widows that have ever exifted
in the world would, in this cafe, be equal to *half*
the number of marriages. And what would take
place in the world muft alfo, on the fame fuppofi-
tions, take place in this fociety.—In other words;
every *other* perfon in fuch a fociety leaving a wi-
dow, there muft arife from it a number of widows
equal to half its own number.——But this does
not determine what number, all living at one and
the fame time, the fociety may expect will come
to be conftantly upon it. For if every widow lived
no more than a year, the fociety would never have
more annuitants upon it than came on in a year.
And on the contrary, if none ever died, the num-
ber of annuitants would go on encreafing for ever.
——'Tis, therefore, neceffary, in order to anfwer
the prefent enquiry, to determine how long the
duration of *furvivorfhip* between perfons of equal
ages will be, compared with the *duration* of *mar-*

b *riage.*

riage. And the truth is, that, fuppofing the pro-
babilities of life to decreafe uniformly (*a*), the for-
mer is equal to the latter; and confequently, that
the number of *furvivors*, (or which is the fame,
fuppofing no fecond marriages) of *widows* and
widowers alive together, which will arife from any
given fet of fuch marriages conftantly kept up, will
be equal to the whole number of marriages; or *half*
of them (the number of widows in particular) e-
qual to half the number of marriages. Now, it ap-
pears that the decreafe in the probabilities of life,
is in fact nearly uniform. According to the *Bre-
flaw*, the *Northampton* and *Norwich* tables of ob-
fervation, almoft the fame numbers die every year
from 20 years of age to 77. After this, in-
deed, fewer die, and the rate of decreafe in the
probabilities of life is retarded. But this deviation
from the hypothefis is inconfiderable; and its ef-
fect, in the prefent cafe, is to render the duration
of

(*a*) That is, fuppofing that out of any given number alive
at any age, the fame number will die every year 'till all are
dead. See the preceding note. That on this hypothefis,
the duration of furvivorfhip is equal to the duration of mar-
riage, when the ages are equal; or, in other words, that
the *expectation* of two joint lives, the ages being equal, is the
fame with the *expectation* of furvivorfhip, may be learnt from
the 18th and 20th problems of Mr. *De Moivre's* treatife on
annuities; and a demonftration of it, together with a parti-
cular explanation of this fubject, may be found at the begin-
ning of the firft effay, at the clofe of Vol. I. of obfervations
on reverfionary payments, to which I muft beg the reader to
turn, if he is at any lofs about the full meaning of what is
here faid.

of furvivorfhip *longer* than it would otherwife be. According to the *London* table of obfervations, the numbers dying every year begin to grow lefs at 50 years of age; and from hence to extreme old age, there is a conftant retardation in the decreafe of the probabilities of life (*b*). Upon the whole, therefore, it appears in anfwer to the prefent queftion, that according to the *three former tables* of " obfervations, and fuppofing no widows to mar- " ry, the number enquired after is *fomewhat great-* " *er* than half the number of the fociety ; but, ac- " cording to the *London Table*, a *good deal greater*."

It muft be carefully remembered, that this has been determined on the fuppofition, that hufbands and their wives are of equal ages, and that in this cafe it becomes an equal chance which fhall die firft. In reality neither of thefe fuppofitions is juft. Hufbands in general are older than their wives; and, in equal ages, the mortality of males has been found to be greater than the mortality of females. For both thefe reafons, it is much more than an equal chance that the hufband will die before his wife, or that the woman will be the furvivor of a marriage, and not the man(*c*). This

b 2   will

(*b*) The reafon of this difference between the *London* and other Tables, is given at the end of the fourth effay, in the Vol. before mentioned.

(*c*) The probability that the woman will be the furvivor in marriage, and not the man, is not lefs than the odds of 3 to 2. See Price Vol. I. p. 82. Among the minifters and profeffors of Scotland it has been more than the odds of 7 to 4. p. 371.

will increafe confiderably the duration of furvivor-
fhip on the part of the woman, and confequently
the number enquired after in this queftion. The
marriage of widows will alfo diminifh this number,
and the operation of thefe caufes will be different
in different fituations. But it is by no means to be
expected (in the fituation of the focieties I have in
view) that the diminution from the latter caufe will
be confiderable enough, to overbalance the operation
of all the other caufes which have been mentioned,
and reduce the number under confideration fo low,
as half the number of marriages (d).

Scholium.

In *London* it appears, that there is a retardation
of the decreafe in the probabilities of life, which
renders the duration of furvivorfhip between two
lives of equal ages, confiderably longer than their
joint continuance. It feems worth obferving, that
this is the reafon why, though the probabilities of
life, and therefore the values of fingle and joint
lives, are lefs in *London* than in other places, yet
the values of reverfions depending on furvivor-
fhips, are in fome cafes greater there. It is proper
to add, that this likewife is the reafon why, in cal-
culating the values of joint lives and reverfions,
the prefent value of an annuity payable yearly to
the furvivor of two equal lives, may come out e-
qual to, or even greater than, the prefent value of
a like

(d) It will be obferved hereafter, that this obfervation has
been found to be true in fact.

a like annuity for the joint lives. As an annuity, during fuch furvivorfhip, will probably not become payable for fome years, and therefore the money given for it will have time to accumulate, it is manifeft, that the value of it could never be equal to the value of an annuity on the joint lives, the payment of which begins immediately, were not the obfervation now made true.

QUESTION III.

" Such a fociety as that defcribed in the pre-
" ceding Queftions being fuppofed; in what time
" will the number of annuitants upon it come to
" a *maximum?*"

ANSWER.

In order to be more clear in anfwering this Queftion, I will firft fuppofe the fociety to comprehend in it from its firft eftablifhment, *all* the married perfons of *all* ages in any town or country, where the number of people continue conftantly the fame. In this cafe, the whole collective body of members will be at their greateft age, at the time of the eftablifhment of the fociety; and the number of members, together with the number of widows left every year, will, taking one year with another, admit of no increafe or diminution. The number of widows in life together, derived from any given number coming on a fociety every year, will increafe continually, 'till as many die off as are added every year; that is, 'till they come to
die

die off as faft as poffible. But they cannot die off as faft as poffible, 'till the whole collective body of widows are at their greateft age; or, 'till there is among them the greateft number poffible of the oldeft widows; and, therefore, not 'till there has been time for an acceffion to the oldeft widows, from the youngeft part of the widows that come on annually.

Let us, for the fake of greater precifion, divide the whole medium of widows that come on every year, into different claffes according to their different ages, and fuppofe fome to be left at 56 years of age, fome at 46, fome at 36, and fome at 26. The widows, conftantly in life together, derived from the firft clafs, will come to their greateft age, and to a *maximum*, in 30 years, fuppofing with Mr. *De Moivre*, 86 to be the utmoft extent of life. The fame will happen to the fecond clafs in forty years, and to the third in fifty years. But the whole body, compofed of thefe claffes, will not come to a *maximum*, 'till the fame happens to the fourth or youngeft clafs; that is, not 'till the end of 60 years. After this, the affairs of the fociety will become *ftationary*, and the number of annuitants upon it of all ages will keep always nearly the fame.

Such is the anfwer to this Queftion, fuppofing a fociety to begin with its complete number of members, confifting of Married perfons of all ages, in the fame proportions to one another, with the proportions in which they exift in the world.— If it begins with its complete number of members, but

but at the fame time admits none above a particular age : If, for inftance, it begins with 200 members all under 50, and afterwards limits itfelf to this number, and keeps it up by admitting every year at all ages between 26 and 50, new members as old ones drop off; in this cafe, the period neceffary to bring on the *maximum* of annuitants will be juft doubled. For, in the firft place, the whole collective body of members will be 60 years in getting to their greateft age, as may eafily appear from whan has been juft faid. The annual medium of widows, therefore, that will come on the fociety will increafe continually for 60 years; it being evident, that the older any fet of married men are, taken one with another, the fafter they will leave widows. And after this annual medium is increafed to a *maximum*, 60 years more will be neceffary to bring to a *maximum* the number in life together, derived from fuch a *fixed* annual medium conftantly coming on.—If fuch a fociety is any number of years in gaining its *maximum* of members, the time neceffary to bring on the *maximum* of annuitants will be ftill further prolonged, and will be equal to twice 60 years with that number of years added.—Moft of the focieties for granting annuities to widows are of this kind; and, therefore, fuppofing them to gain their complete number of members in ten years, and for ever afterwards to preferve it, the number of annuitants upon them will go on increafing for 130 years.————It is proper, however, to be remembered, that the increafe will be quicker at firft, and afterwards flower; and that, within 20 or 30

years

years of the end of this term, it will be fo flow as fcarcely to be fenfible, though ftill real.

All who will beftow due attention on this fubject muft fee thefe decifions to be juft; and a demonftration of them might be given, in a form more ftrictly mathematical, were it neceffary.

From hence it appears it is vain to form fuch eftablifhments with the expectation of feeing their fate determined foon by experience. If not more extravagant than any ignorance can well make them, they *will* go on profperoufly for 20 or 30 years; and, if at all tolerable, they *may* fupport themfelves for 50 or 60 years; and at laft end in diftrefs and ruin. No experiments, therefore, of this fort fhould be tried haftily. An unfuccefsful experiment muft be productive of very pernicious effects. All inadequate fchemes lay the foundation of *prefent* relief on *future* calamity, and afford affiftance to a *few* by difappointing and oppreffing *multitudes* (*a*).

Question IV.

" Suppofe the members of fuch a fociety as
" that defcribed in the preceding queftions, to
" chufe making *annual payments during the conti-*
" *nuance of marriage*, in lieu of the fum which
" the

(*a*) Dr. Price, Vol. I. page. 105.

" the reverſionary Annuity for their Widows is
" worth in *preſent money*: What ought theſe *Annual*
" *Payments* to be, eſtimating intereſt at 4 *per cent* ?"

ANSWER.

This will be eaſily determined, by finding what
Annual Payments, during two joint lives of given
ages, are equivalent to the value of the reverſion-
ary annuity in *preſent money*.—Suppoſe as in Queſ-
tion I. the two joint lives to be each 40, and the
reverſionary annuity 30*l. per annum*. An annual
payment during the continuance of two ſuch
lives is worth, according to Dr. *Halley*'s Table of
Obſervations, 9.82 (*a*) years purchaſe. The an-
nual payment then ought to be ſuch as being mul-
tiplied by 9.82, will produce, (*b*) *l.* 101.1, the
preſent value of the annuity in one payment by
Queſtion I. Divide then *l.*101.1 by 9.82, and the
quotient, or *l.* 10.3 will be the anſwer.——This is

c   very

(*a*) See Table II.

(*b*) Particular notice ſhould be taken of the method of no-
tation here uſed, becauſe it will be carried through the whole
of this work.——The figures on the right hand of the full-
point, ſignify the decimal parts of 1*l.* Thus; *l.*101.1, is
*l.*101 and the 10th of 1*l.* or *l.*101 and 2*s.*——*l.*9.39, is *l*9,
and 39 hundredths of 1*l* or *l.*9: 7*s*: 10*d.*——*l.*11.33, is *l.*11,
and   hundredths of 1*l* or *l.*11: 6*s*: 7*d.*——In general; it
ſhould be remembered, that two ſhillings allowed for every
unit in the *firſt* place of decimals, and two-pence half-penny
for every unit in the *ſecond* place of decimals, will give,
nearly enough, the value of the decimal part of every ſuch
expreſſion.

very nearly the annual payment of all the members at an average, fuppofing equal numbers to offer themfelves for admiffion of every age between 30 and 50. As much as fome give lefs, others ought to give more, according to their excefs of age. Thus, the annual payment of a married perfon, 30 years of age, ought to be $l.9.39$; and of a perfon 50 years of age $l.11.33$——If the values of joint lives and of the reverfionary annuity are taken agreeably to the *London* Table of Obfervations, thefe annual payments will be, for 30 years of age, $l.10.9$—for 40, $l.12.5$,—for 50, $l.14.5$.

If either the rate of intereft is fuppofed lower, or wives are fuppofed younger than their hufbands, the annual payments will be increafed. But there is no occafion for pointing out particularly the difference. It may be eafily found in any cafes by the directions now given. There is, however, one obfervation which ought to be here carefully attended to.—This method of calculation fuppofes, that the firft annual payment is not to be made 'till the *end* of a year. If it is to be made *immediately*, the value of the joint lives will be increafed one year's purchafe; and, therefore, in order to find in this cafe the annual payments required, the value in prefent money found by Queft. 1. muft be divided by the value of the joint lives increafed by unity, and, in this way, the preceding values at 4 *per cent.* according to the *Breflaw* Obfervations, will be found to be $l.8.62$—$l.9.35$—$l.10.07$—According to the *London* obfervations, $l.10$,—$l.11.2$, —$l.12.7$.

QUESTION V.

" A fociety may chufe to make abatements in
" thefe annual payments, and to require the re-
" mainder of the value of the reverfionary annui-
" ty to be given, in fines or premiums at the time
" of admiffion. It may, for inftance, chufe to fix
" the annual payment of all the members to 5
" guineas. What, in this cafe, would be the
" premium due at admiffion, the annuity being
" fuppofed 30*l.* *per annum,* and intereft being at
" 4 *per cent ?*"

ANSWER.

From the whole prefent value of the annuity in
one payment, fubtract the value of 5 guineas *per
annum,* during the joint lives; and the remainder
will be the anfwer.

Suppofing the joint lives both 40, the whole
prefent value of the annuity in one payment is,
according to the *Breflaw* Obfervations, *l.*101.1, by
Queft. I.——The value of 5 guineas *per annum,*
or of *l.*5.25 *per annum,* during two fuch joint lives,
is *l.*5.25, multiplied by the value of the joint
lives; that is, 5,25, multiplied by 9.82, or
*l.*51.55; and this fubtracted from 101.1, gives
*l.*49.5, the anfwer required for two lives at the
age of 40 ——The anfwer found in the fame way
for two lives whofe common age is 30, is *l.*46.5,—
and for two lives at 50, 50*l.*

Ac-

According to the *London* Obfervations, thefe values are, for two lives at 30, *l.*54.6——At 40, *l.*59.4.——At 50, *l.*63.3.

If the firft of the annual payments is to be made *immediately*, the true anfwer will, in every inftance, be the values found in the manner now directed, diminifhed by the annual payment; or, in the prefent cafe, 5 guineas lefs than the values fpecified.

The values, in *premiums* and *annual payments*, of any other reverfionary annuity, will be as much greater or lefs than thefe, as the annuity itfelf is greater or lefs.

QUESTION VI. (*a*)

" Let the fcheme of a fociety for granting an-
" nuities to widows, be, that if a member lives
" *a year* after admiffion, his widow fhall be enti-
" tled to a life annuity of 20*l.* If *feven* years, to
" 10*l.* more, or 30*l.* in the whole. If *fifteen*
" years, to another additional 10*l.* or 40*l.* in the
" whole. What ought to be the annual payments
" of the members for the ages of 30, 40, and 50,
" fuppofing them of the fame ages with their
" wives, and allowing compound intereft at 4
" *per cent* ?"

ANSWER.

(*a*) Queft. VIII. in Dr. Price, Vol. I.

ANSWER.

According to the *hypothefis*, explained p. 4; and, therefore, very nearly, according to the Tables of Obfervation for *Breflaw*, *Norwich*, and *Northampton*,

$$l.8.44—l.8.69—l.9.05.$$

According to the *London* Obfervations,

$$l.9.41—l.10.17—l.10.92.$$

Thefe values are eafily deduced from the values in the laft Queftion. For example : The value of 10*l. per annum* for life to 40 after 40 (*b*), provided the joint lives do not fail in *one* year, is, according to the *hypothefis*, *l*. 30.33. The value of 20*l. per annum*, in the fame circumftances, is, therefore, *l*.60.66.——In like manner, the value of 10*l.* after *feven* years, is *l*.17.44. And of 10*l.* after 15 years, *l*.7.3.——Thefe values together make *l*.85.4, or the value of the expectation, defcribed in this Queftion, in a *fingle prefent payment*; which, divided by 9.82, (the value by Table II. of two joint lives at 40) gives *l*.8.69, the value of the fame expectation in *annual payments*, during the joint lives——In the fame manner may be found the anfwer in all cafes to any Queftions of this kind.

Thefe

(*b*) That is, to the furvivor of two perfons each of whom is 40 years of age.

Thefe calculations fuppofe, that the annual pay-ments do not begin till the *end* of a year. If they are to begin *immediately*, the true *annual payments* will be, as was before obferved, the *fingle* pay-ments, divided by the value of the joint lives in-creafed by unity; and in the prefent cafe they will be, by the *hypothefis*,

$$l. 7.75 - l.7.9 - l.8.07.$$

By the *London* Obfervations,

$$l.8.52 - l.9.06 - l.9.51.$$

By the method of calculation now explained, may be eafily found in all cafes, fuppofing the an-nual payments previoufly fettled, what the rever-fionary annuities are, correfponding to them in va-lue.——Thus, the annuities being the fame with thofe mentioned in this Queftion, the *mean* annual payments for all ages between 30 and 50, are nearly 8*l.* according to the *higheft* probabilities of life; 9*l.* according to the *loweft*; and 8 guineas the *medium* (*c*); intereft being at 4 *per cent.* and the firft payment to be made immediately.

If

(*c*) The value of this expeƈtation, fuppofing married men 40 years of age, and their wives 30, is, in a *fingle* pay-ment, 113*l.* In annual payments beginning immediately *l.*9.88, by the *hypothefis.*——And 107*l.* and *l.*10,93, by the *London* Obfervations.

If the mean annual payments, beginning imme-
diately, are fixed to five guineas, the correspond-
ing life annuities will be nearly (by the *hypothesis*)
12*l.* if the contributor lives a year, and 24*l.* if he
lives 7 years; or (by the *London* Obfervations) 12*l.*
if he lives a year, and 20*l.* if he lives 7 years *(d)*.

It is observable, that the difference in the va-
lues of the annuities, arising from difference of
ages, and the difference of the probabilities of
life, is less in this Question than in Question 4th;
and that, consequently, the plan proposed in it,
is the safeft as well as the moft equitable and en-
couraging, that a society can adopt.

It

(d) If the annuities in expectation are 14*l* provided a mem-
ber lives a year, and 20*l.* provided he lives feven years, the
proper *mean fingle* payments for all ages, taken one with ano-
ther, under 50 or 52, is 50 guineas nearly, according to all
the Tables of Obfervation, fuppofing *equality* of age between
men and their wives And the addition which ought to be
made, on account of excefs of age on the man's fide is, tak-
ing the neareft and the eafieft round fums, about a guinea
and $\frac{1}{2}$ for every year as far as 17 years; or, in the annual pay-
ments,(fuppofed a 5 guineas) $\frac{1}{2}$ a guinea *per annum* for five years,
excefs, and $\frac{1}{2}$ a guinea more for every four years excefs beyond
five years, 'till the excefs comes to be 17 years. And I be-
lieve, that 60 guineas in *fingle payments*, and fix guineas in *an-
nual payments* beginning immediately, may be very well ftated
at the *loweft common* payments proper to be required, fuppo-
fing all married men under 52, taken into a fociety, without
enquiring into the difference of age between them and their
wives, the annuities being all along fuppofed to be *life* annui-
ties, and intereft reckoned at 4 *per cent.*

It is neceffary to remark here further, that *yearly* payments which begin immediately, are more advantageous than *half-yearly* payments which begin immediately. In an Effay publifhed in the Philofophical Tranfactions, Vol. 66, p. 109, and inferted in the fecond Volume of the Obferv-ations, I have fhewn, that, in the cafe of life annuities, *half-yearly* payments, which begin at the end of half a year, are nearly a fifth of a year's purchafe better than *yearly* payments, which begin at the end of a year. And it is manifeft, that *half-yearly* payments, which begin *immedi-ately*, are no more than half a year's purchafe better than thofe which begin at the *end* of half a year. But *yearly* payments, which begin *im-mediately*, are a *whole year*'s purchafe better than the fame payments to begin at the *end* of a year. The difference of value, therefore, between *year-ly* and *half-yearly* payments, fuppofing both to begin immediately, is three-tenths of a year's purchafe in favour of the former —The whole of this fub-ject may be feen accurately ftated in the Effay juft referred to.

QUESTION VII. (a)

" Suppofe an inftitution for the relief of wi-
" dows to extend its affiftance likewife to the fa-
" milies of married men, provided they leave no
widows

(a) Quft. xii Dr. Price, Vol. I.

" widows, Suppose, for inftance, that in this
" cafe children are to be entitled to 100*l.* What
" is fuch an expectation worth, in prefent pay-
" ment, according to Dr. *Halley*'s Table, intereft
" being at 4 *per cent.?*"

Answer.

If 40 is the mean age at which members are ad-
mitted on fuch an inftitution, and 32 the mean
age of their wives, the anfwer (fuppofing no fub-
fequent marriages) is, by the 33d Problem in Mr.
Simpfon's Select Exercifes, p. 298, and the cor-
rections already explained, *l.* 13.80, (*b*) taking the
expectation and values agreeably to Mr. *De
Moivre*'s hypothefis.

d   But

(*b*) This Problem and its folution are given by Mr. *Simpfon* in
the following words. " A and his heirs are entitled to an eftate
" of a given value, upon the deceafe of B, provided B furvives
" A ; to find the value of their expectations in *prefent* money."
—Solution. " Find the value of an annuity on the longeft
" of two equal lives, whereof the common age is that of
" the older of the lives A and B ; which value fubtract from
" the perpetuity, and take half the remainder ; then it will
" be, as the expectation of duration of the younger of the
" lives A and B is to that of the older, fo is the faid half re-
" mainder to the number of years purchafe required, when
" the life B *is the older of the two.* But if B *be the younger*;
" then to the number thus found, add the value of an an-
" nuity on the longeft of the lives A and B, and fubtract
" the fum from the perpetuity, for the anfwer in this cafe,"

If

But there is a reduction neceſſary, on account of the chance there is, that a widower may marry again. Suppoſe, therefore, one half of all widowers to marry a ſecond and third time, and that two fifths of ſuch widowers ſurvive theſe ſubſequent marriages. In this caſe, $\frac{1}{2}$ added to $\frac{2}{5}$ of $\frac{1}{2}$, or $\frac{7}{10}$ of all who become widowers, will die without leaving widows and therefore $\frac{7}{10}$ of l. 13.8, or l. 9.66, will be the anſwer. If only *one fourth* of all who become widowers marry again, and two-fifths of theſe ſurvive, the anſwer will be l. 11.73.

This calculation ſuppoſes all marriages to leave children who ſurvive their parents. If this is conſidered as uncertain, the values now determined muſt be diminiſhed in the proportion of this uncertainty.—Thus; if one marriage in ſeven fails of leaving

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If the eſtate is 4l. *per annum*, the age of B 40, and of A 32, intereſt 4 *per cent.* the anſwer by this rule comes out l. 14.35, which divided (as in the preceding Queſtion) by 104, gives l. 13.80, the value as above, of 100l. in money. If B is 30 and A 40, the ſame value is 20l.

N. B. The value ofthe longeſt of two lives is always the *difference* between the value of the *joint* lives, and the *ſum* of the values of the two given *ſingle* lives. Thus; the value of a live at 40, is, by Table I, 13.2. The *ſum* of the values of two ſuch lives, is 26.4. The value of two joint lives, whoſe common age is 40, is, by Table II, 9.82; and the difference is 16.58. or the value of the *longeſt* of two lives at 40.

leaving children (*c*) that furvive their parents; thefe values will be reduced a *feventh* part or to *l*. 8.28, if *half*, and *l*. 10.05, if a *quarter* of all widowers marry.

In this way may any other queftions of the fame kind be anfwered on any fuppofitions that may be thought moft reafonable.

QUESTION VIII (*a*).

" Let an eftablifhment be fuppofed which takes
" in at once all the marriages in a country, or all
" marriages among perfons of a particular profef-
" fion within a given diftrict, and fubjects them
" for perpetuity to a certain equal and common
" tax, or annual payments, in order to provide
" life annuities for fuch widows as fhall refult from
" thefe marriages. What ought the tax to be,
" fuppofing the annuity 20*l*. and calculating at 4
" *per cent*. from Mr. *De Moivre*'s valuation of
" lives ?

e 2   Anfwer

(*c*) This for many years has been nearly the fact among the minifters and profeffors in *Scotland*.

(*a*) Queft. xiii. Dr. Price Vol I.

ANSWER.

Since at the commencement of fuch an eſtabliſhment, all the oldeſt, as well as the youngeſt marriages are to be entitled equally to the propoſed benefit, a much greater number of annuitants will come immediately upon it, than would come upon any ſimilar eſtabliſhment, which limited itſelf in the admiſſion of members to perſons not exceeding a given age. This will check that accumulation of money, which ſhould take place *at firſt*, in order to produce an income equal to the diſburſements at the time when the number of annuitants comes to a *maximum*; and, therefore, will be a particular burthen upon the eſtabliſhment in its infancy. For this, ſome compenſation muſt be provided and the equitable method of providing it, is, by levying *fines* at the begining of the eſtabliſhment, on every member *exceeding* a given age, proportioned to the number of years which he has lived beyond that age. But in the preſent queſtion, it is ſuppoſed, that fuch fines cannot be conveniently levied, or that every payment muſt be equal and common, whatever diſparity there may be in the value of the expectations of different members. The fines, therefore, muſt be reduced to one common one, anſwering as nearly as poſſible to the diſadvantage I have mentioned, and payable by every member at the time when the eſtabliſhment begins. After this, the eſtabliſhment will be the ſame with one that takes upon it all *at the time they marry* (*b*); and the tax or annual

payment

(*b*) *That is, in which all the members become ſubject to payment of the tax immediately after marriage.*

payment of every member adequate to its fupport, will be the annual payment *during marriage* (*c*), due from perfons who marry at the mean age at which, upon an average, all marriages may be confidered as commencing.——There are then two points to be here determined: The *fines* neceffary to be paid at firft, according to the account I have juft given; and the *conftant annual payment* neceffary to be made by every member, as an equivalent for the expectation provided by the eftablifhment.——The *fines* to be paid at firft are, for every particular member, the fame with the difference between the value of the expectation to him at his prefent age, and what would have been its value to him had the fcheme began at the time he married? Or, they are for the whole body of members, the difference between the value of the common expectation, to perfons at the mean age of all married perfons taken together as they exift in the world, and to perfons at that age, which is to be deemed their mean age when they marry.

Thus; let 33 for the man, and 25 for the woman, be the mean ages of all that marry annually. Let alfo 48 be the mean age of all the married men in the world, and 40 of married women (*d*).——Now,

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(*c*) *That is, the annual payment of every member* during life *will be equal to the annual payment* during marriage *which ought to be paid by perfons* who marry, &c.

(*d*) When I here call 48 the mean age of all married men, and 40 the mean age of married women, I do not intend to fuppofe, that there are as many married perfons who exceed
<div align="right">thefe</div>

ANSWER.

Since at the commencement of such an establishment, all the oldest, as well as the youngest marriages are to be entitled equally to the proposed benefit, a much greater number of annuitants will come immediately upon it, than would come upon any similar establishment, which limited itself in the admission of members to persons not exceeding a given age. This will check that accumulation of money, which should take place *at first*, in order to produce an income equal to the disbursements at the time when the number of annuitants comes to a *maximum*; and, therefore, will be a particular burthen upon the establishment in its infancy. For this, some compensation must be provided and the equitable method of providing it, is, by levying *fines* at the begining of the establishment, on every member *exceeding* a given age, proportioned to the number of years which he has lived beyond that age. But in the present question, it is supposed, that such fines cannot be conveniently levied, or that every payment must be equal and common, whatever disparity there may be in the value of the expectations of different members. The fines, therefore, must be reduced to one common one, answering as nearly as possible to the disadvantage I have mentioned, and payable by every member at the time when the establishment begins. After this, the establishment will be the same with one that takes upon it all *at the time they marry* (b); and the tax or annual payment

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(b) *That is, in which all the members become subject to payment of the tax immediately after marriage.*

payment of every member adequate to its support, will be the annual payment *during marriage* (c), due from persons who marry at the mean age at which, upon an average, all marriages may be considered as commencing.——There are then two points to be here determined : The *fines* necessary to be paid at first, according to the account I have just given ; and the *constant annual payment* necessary to be made by every member, as an equivalent for the expectation provided by the establishment.—— The *fines* to be paid at first are, for every particular member, the same with the difference between the value of the expectation to him at his present age, and what would have been its value to him had the scheme began at the time he married ? Or, they are for the whole body of members, the difference between the value of the common expectation, to persons at the mean age of all married persons taken together as they exist in the world, and to persons at that age, which is to be deemed their mean age when they marry.

Thus ; let 33 for the man, and 25 for the woman, be the mean ages of all that marry annually. Let also 48 be the mean age of all the married men in the world, and 40 of married women (d).
——Now,

(c) *That is, the annual payment of every member* during life *will be equal to the annual payment* during marriage *which ought to be paid by persons* who marry, &c.

(d) When I here call 48 the mean age of all married men, and 40 the mean age of married women, I do not intend to suppose, that there are as many married persons who exceed
these

——Now, he that will calculate for thefe ages, in the manner directed in Queft. IV. will find, that the value in *annual payments* during marriage, and begin-

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thefe ages, as there are who fall fhort of them. It is likely that the latter are moft numerous; and it is neceffary that this fhould be the cafe, to render the fuppofition I make juft. —If all marriages commenced at 33 for the man, and 25 for the woman, one half of them would be diffolved by the time the men were 50, and the women 42; for (by the *Hypothefis*, and alfo nearly by the *Breflaw*, *Norwich* and *Northampton* tables) there is an equal chance for the joint continuance of two lives, whofe ages are 25 and 33, *feventeen* years. Forty two and fifty then would be properly the mean ages at which widow-hood would commence; meaning by thefe, " the ages " on each fide of which equal numbers are left widows and " widowers "——But, tho' in this cafe half the marriages of every year would be diffolved in 17 years, they would not be *all* diffolved in twice that time. So far would this be from happening, that about a 7th part would continue beyond twice 17 years; nor would it be *certain*, that they would be all diffolved till near the extremity of the poffible extent of life. Tho', therefore, an equal number of marriages would be diffolved, or an equal number of widows and widowers left *before* 50 and 42, and *afterwards*, yet the ages of the latter would, one with another, much more exceed 50 and 42, than the ages of the former (that is, of the widows and widowers left *before* 50 and 42) would fall fhort of them. And the number of marriages alfo in the world, among perfons of greater ages than thefe, would be much fewer than among perfons of leffer ages ——In other words: the period, at which the marriages that have been contracted are half diffolved is not the period at which the number of marriages conftantly exifting is equally divided, but this period falls fome years fooner; and the period I have in view, falls in that part of the interval between thefe two periods, where the greater ages of the marriages on one fide, are juft enough to compenfate (in fuch a calculation as that I have given) their deficiencies in number, compared with the number of marriages on the other fide. See Dr. Price, vol. II. additional Effays and Notes, page 68.

beginning immediately, of the expectation of an annuity of 20*l. per annum*, by a person 25 years of age, after a life whose age is 33. is *l.*6.64.— And that *l.*8.04, is the value of the same expectation, the ages being 48 and 40.

The former, therefore, is the payment for perpetuity from every member of the establishment; and the value of the *difference* between it and the latter, or of *l.*1.4 *per ann.* payable during two joint lives, whose ages are 40 and 48, that is, *l.*14.2, is the fine necessary to be levied on every married member at the beginning of the establishment (*e*).

It would be easy to extend the benefit of such an establishment, so far as to provide 100*l.* for the children of members, provided they leave no widows; and the necessary addition on this account to the perpetual annual payments, can scarcely, in the circumstances this question supposes, be much more than about 15*s.* payable during life, and excluding from all benefit such as happen to be widowers

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(*e*) An annuity for ever, the first payment of which is to be made *immediately*, is worth 26 years purchase, interest being at 4 *per cent.* *l.*14.2 therefore, is equivalent in value to 0.55*l.* or 11*s. per annum*, for ever. Add this to *l.*6 64, and it will appear, that *l.* 7.19. that is 7*l.* 4*s.* nearly *per annum*, beginning immediately is the answer to this Question, supposing the value of the *fine* to be provided for in the perpetual annual payments.

dowers at the commencement of the eftablifhment, and do not afterwards marry (*f*).

If, in fuch an eftablifhment, all perfons of a particular denomination, whether married men widowers, or batchelors, are fubjected alike to the taxes and fines; they ought to be as much *lefs*, as the whole number of perfons fubjected to them, is *greater* than the number of marriages conftantly exifting.

In carrying thefe fchemes into execution, there cannot be a more eafy or equitable way of raifing the neceffary fines, than by providing, that none fhall be entitled to any expectation for a few of the firft years. Thus; an eftablifhment, entitling widows to 20*l. per annum* for life, and confifting of 667 married members, and 344 unmarried(*g*) always kept up at an average, ought to begin with a capital of *l.*14.2 multiplied by 667, or 9471*l.* befides one payment in hand of the conftant annual payments.

(*f*) *The Bill lately brought into parliament for eftablifhing a fund for the relief of the widows and children of the Clergy of Ireland, not only provided that the children even of fuch perfons as above defcribed, provided faid children fhall on June, 24, 1784, be under 16 years of age, fhall receive* 200*l. in cafe the rate to be chofen by the Father fhall be correfpondent to an annuity of* 20*l. to be enjoyed by his widow, had he left one, but alfo provided that if any widow entitled to an annuity of* 20*l. fhall die before fhe has received* 200*l. her children under* 16 *fhall receive in one fum as much as added to what fhe has received fhall make up* 200*l.*

(*g*) *That is Batchelors and widowers.*

payments. That is, (the proper annual payment of every member being in this cafe $\frac{667}{1011}$, multiplied by $l.6.64$, that is $l.4.38$) it ought to begin with a capital of $13,889l.$ over and above the payment of $l.4.38$, at the *end* of every year for ever afterwards (f)——The exclufion of all the firft members from any benefit, unlefs they furvive the firft *two* years, or live to make *three* payments, would raife this capital nearly. And fuch an exclufion for *three* or *four* years, would be an advantage fo confiderable, that it would probably give fecurity and ftability to the fcheme for all fubfequent time.

In thefe obfervations, I have had in view fome fchemes, which have been eftablifhed in England ; but more particularly, one eftablifhed by act of parliament among the clergy in *Scotland*; of which, I fhall have occafion to take further notice.

e   I have

(f) Or, fuppofing the value of $9471l.$ (the fine) provided for in the annual payments, it ought to receive every year, at the *beginning* of the year, a contribution from each member of $l.4.74.$

The want of fuch a capital is compenfated in the fcheme propofed to be eftablifhed by the bill before mentioned, by the additional payments to which members are fubjected in refpect of their marriage, or their being when they firft become members forty years of age or upwards——and by the deductions it requires to be made from the annuities of widows, whofe hufbands payments have not been equal to three years of their annuities.

I have chosen to calculate here only from Dr. *Halley*'s Table, or Mr. *De Moivre*'s *hypothesis* grounded upon it, because the *London* Table is, by no means, adapted to the cases in view.

The difference of eight years between the ages of men and their wives, as here taken, is probably too little; and for this reason, and also on account of the greater mortality of males, the values I have given should be considered as the *lowest* that any scheme ought to provide.

It should be further remembered, that when the mean ages, at which marriages commence, are supposed to be 33 and 25, all *second* and *third* marriages are included; and that it is to be expected, that almost all *these* marriages will begin after these ages; and likewise, that a considerable proportion of the first marriages will begin a much longer time *after* these mean ages, than any of the other first marriages will begin before them.——Probably, therefore, these mean ages should not be taken younger. One or two years, however, more or less, in every supposition I have made, will make no difference of any consequence.

(*g*) The preceding observations have gone on the supposition, that the reversionary annuities are to be *for life*. What difference in favour of these societies arises from the circumstance, that
the

(*g*) Dr. Price, vol. I. p. 78.

the annuities are to be paid only *for widowhood*, cannot be exactly determined. Some judgment, however, may be formed of it from what has been said at the conclusion of Queft. II. But in the circumftances of thefe focieties, it cannot be expected, that above one in 10, or perhaps one in 20, will marry. The perfons moft likely to enter into them, are fuch as have not the profpect or ability of making competent provifions for their widows in other ways. The widows left, therefore, will in general be unprovided for, and, being alfo left with families of children, it is quite unreafonable to expect, that any confiderable proportion fhould marry. This is true even of fuch as may happen to be left *young*.—Moreover, the profpect of the lofs of their annuities will have a particular tendency to check marriage among them.—(*h*) For all thefe reafons it feems to me likely, that the benefit, which thefe focieties will derive from marriages among their annuitants, will not be very confiderable; or at leaft not *fo* confiderable as to be equal to the advantages I have allowed them by calculating on the fuppofitions, that the money they

e 2   receive

(*h*) To remedy in fome degree this inconvenience the bill before mentioned provides, that if an annuitant fhall marry a perfon fubject to a rate, fhe fhall receive half her annuity 'till her after-marriage fhall be diffolved—If fhe fhall be the furvivor fhe is of courfe to receive an annuity proportioned to her laft hufband's rate—It is alfo provided that if fhe fhall marry a perfon not fubject to a rate, fhe fhall after the diffolution of the marriage, if fhe be the furvivor, be reftored to her annuity.

It may be proper here to obferve, that this note as well as the former notes printed in Italics, were added by the Editor.

receive will be *always improved perfectly, without loss or delay, at the rate of 4 per cent. compound interest*; that the probabilities of life among males and females are the same, and all husbands likewise of the same ages with their wives, and that consequently the *maximum* of widows on such societies can amount to no more than half the number of marriages (*i*)——With respect to the last of these suppositions, it deserves to be particularly observed, that by an enquiry made some years ago in *Scotland*, it was found, that the widows of the *ministers* and *professors* there (*k*), notwithstanding the diminution occasioned by their marrying, did exceed half the number of marriages. And certainly it would be unreasonable in other societies not to reckon that the same will happen among them.——Indeed it

seems

(*i*) Care should be taken in these societies, not to judge of the proportion of widows that will marry, from the proportion that may happen to marry during their first years. For most of the widows that will be left at first will be young ; whereas the greater part will not be young when they commence widowhood, after a society has subsisted 30 or 40 years ; and therefore, tho' one in 3 or 4 should marry at first, it will not be reasonable to expect, that half so many should marry after the affairs of the society become stationary.

(*k*) The number of married ministers and professors, for 17 years from 1750 to 1766 was at a medium 667. And from the enquiries that have been made, it appears, that from this whole body near 400 widows constantly living are derived.

feems certain that, notwithftanding the hazards that attend child-bearing, the probability, that the woman fhall furvive in marriage, and not the man, is much greater (*l*) than is commonly imagined. It has been fhewn elfewhere, that it is not lefs than the odds of 3 to 2 (*m*), and had I calculated agreeably to this fact, the values of annuities for widows, would have been given near a quarter greater than they have been given on the fuppofition, that the chance of furvivorfhip is equal between men and their wives.—It muft be added, that I have made no account of any expences attending the execution and management of the fchemes of thefe focieties. Some expences there muft be, and fome advantages fhould be always provided in order to compenfate them.

There are in this Kingdom *(England)* many inftitutions for the benefit of widows: in general,

as

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derived. The number of widows left annually has for the laft 36 years been $19\frac{1}{16}$; and, for 10 years, ending in the year 1767, but nine of thefe had married ——Of the annuitants likewife (about 160 in number) on the fund eftablifhed among the Diffenters in *London*, for relieving the widows of indigent minifters, it is fouud that few ever marry.

(*l*) Partly, on account of the greater mortality of males, but chiefly on account of the excefs of age on the man's fide.

In March 1780, 167 hufbands had died in the *Laudable Society*, and only 138 wives.

(*m*) Among the minifters and profeffors of *Scotland* it has been more than the odds of 7 to 4.

as far as I had any information concerning them, they are founded on inadequate plans, having been formed juft as fancy dictated, without any knowledge of the principles on which the values of reverfionary annuities ought to be calculated. The motives which influence the contrivers of thefe inftitutions may be laudable, but they ought I think to have informed themfelves better.

It may be faid in defence of focieties which have adopted fuch inftitutions that deficiencies in their plans, cannot be of much confequence, if a conftant equality be preferved between their income and expences, by reducing the annuities as there fhall be occafion : And from hence it is inferred, that they can never be in any danger of a bankruptcy. But in anfwer to this it is to be obferved, that the time when they will begin to feel deficiencies is fo diftant, that it will be too late to remedy paft errors, without finking the annuities fo much, as to render them inconfiderable and trifling. All that is given too much to *prefent* annuitants is fo much taken away from *future* annuitants. And if a fcheme is *very* deficient, the firft annuitants may, for 30 or 40 years receive fo much more than they ought to receive, as to leave little or nothing for any who come after them. Deficient fchemes, therefore, are attended with particular injuftice; and, this injuftice will be the fame, if, inftead of *reducing* the annuities, the annual payments fhould be *increafed*; for all the difference this can make will be, to caufe the injuftice to fall on *future contributors*, inftead of *future annuitants*.

But

But what requires moſt to be conſidered here is that, after either the annuities have been for ſome time in a ſtate of reduction, or the contributions in a ſtate of increaſe, it will be ſeen that theſe Societies have gone upon wrong plans, and, therefore, they will be deſerted and avoided ; the conſequence of which will prove ſtill greater deficiencies in their annual income, and a more rapid deſertion and decline, 'till a total diſſolution and bankruptcy take place.—This will be the death of moſt of the preſent ſocieties for providing for widows, if they continue to be encouraged, and do not ſoon alter their plans: And at that period the number of *annuitants* will be greater than ever ; whoſe annuities, having no other ſupport than the poor remains of a ſtock always inſufficient, will be ſoon left, without the probability of relief, to lament that ignorance and credulity which gave riſe to theſe ſocieties, and which had ſo long ſupported them.

(*n*) The general diſpoſition, which has lately ſhewn itſelf to encourage annuity ſocieties, is a matter of the moſt ſerious concern , and ought I think to be taken under the notice of the Legiſlature ſhould ſuch ſocieties continue much longer deaf to the calls of juſtice and humanity.

Firſt. They are laying (as I have proved) the foundation of much future miſchief ; and no government ought to ſee this with a careleſs eye.

Secondly.

(*n*) See Dr. Price's Obſervations, &c. Page 310. 3d Edition.

Secondly. The principle by which they are upheld is bafe and iniquitous. The *prefent* members believe that the fchemes they are fupporting will laft their time, and that they fhall be gainers; and as for the injury done to their fucceffors, it is at a diftance, and they care little about it.———In conformity to this principle, the founders of thefe focieties begin low; *fo* low, as not to require perhaps, a *fourth* or *fifth* of the values of the annuities they promife. Afterwards; they advance gradually, juft as if they imagined, that the value of the annuities was nothing determinate, but increafed with every increafe of the fociety. But, as no ignorance can believe this, the true defign appears to be, to form foon as large a fociety as poffible, by leading the unwary to endeavour to be foremoft in their applications, left the advantage of getting in on the eafieft terms fhould be loft.— It is well known, that thefe arts have fucceeded wonderfully, and that, in confequence of them, thefe focieties now confift of perfons who, for the *fame* annuities, make higher or lower payments according to the time when they have been admitted; and the generality of whom, therefore, muft know, that either more than the values have been required of the members laft admitted; or if not, that they are themfelves expecting confiderable annuities, for which they have given no valuable confideration, and which, if paid them, muft be ftolen from the pockets of fome of their fellow-members. What fcenes, therefore, of *difhonefty* on the one hand, and of unhappy *credulity* on the other, are thefe focieties?

Thirdly,

Thirdly. There are many honeſt men in theſe ſocieties who, having thro' miſinformation, had the misfortune to enter into them, now repent, and would be glad to withdraw. |But, having made conſiderable payments which they cannot get back, they are forced to go on with further payments, in order to avoid loſing all their former ones. Theſe perſons wiſh for aſſiſtance from the legiſlature; and their caſes, I think, require aſſiſtance.

OF THE

ESTABLISHMENT

AMONG THE

MINISTERS in SCOTLAND.

Price on Reverſionary Payments, vol. I. page 70.

There is one aſſociation of particular conſequence, of which it is neceſſary I ſhould take notice. I mean the *aſſociation* among the miniſters and profeſſors in *Scotland*, for making proviſions for their widows and orphans. Several aſſociations

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in *England* (*a*) have been formed on the model of this eftablifhment; and the fuccefs with which it has been hitherto attended, has been the principal caufe to which they have owed their rife.—It is therefore proper I fhould give fome account of it—and it will be fufficient with this view to mention, that for, "an annual payment, which " began *immediately*, of five guineas from 1011 " contributors, 667 of whom are married perfons; " befides a tax on weddings producing about 142*l.* " *per annum*; it entitles every widow to an annuity " of 20*l.* during widowhood; and alfo, every fa- " mily of children that fhall be left by fuch mem- " bers as die without leaving widows, to 200*l.*"

This fcheme contains a variety of other particulars; but this is its fubftance———It commenced on the 25th of March, 1744; and from that time, to the 22d of November, 1770 (*b*), or in 26 years and near 8 months, 151 Minifters and Profeffors died, and left 151 families of children without widows; that is 5,66 fuch families were left annually;

(*a*) There is one among the Diffenting Minifters in the counties of *Chefter* and *Lancafbire*, and another among the Diffenting Minifters in *Cumberland*, *Northumberland*, *Weftmoreland*, and *Durham.*——Even the *London Annuity* Society, tho' its plan is totally different, profeffes to form itfelf on the principles of the *Scotch* eftablifhment, and to derive encouragement from it.

(*b*) In Nov. 1779, or 35 years and 8 months, 199 minifters and profeffors had died, and left 199 families of children without widows; that is 5.58 annually.

ally; and the annual difburfements to them have therefore, been 1132*l.* fubtract this fum from 5450*l.* the whole annual income; and the remainder, or 4318*l. per annum,* will be the ftanding provifion for bearing the expence of all the annuitants poffible to be derived from 667 marriages. Such an annual payment, or 4.27 each from 1011 contributors, is the fame with 6.55 each from 1011 contributors; and, confequently it appears, that in this eftablifhment a contribution is received equivalent to an annual payment, beginning *immediately,* of 6.55, from every married man, in order to entitle his widow to an annuity of 20*l.* during her widowhood.

It may be obferved that the annual income for the fupport of this eftablifhment, fuppofing it to have only the benefit of widows in view, ought to be *l.*7.19 *per annum,* from every marriage, according to Queft. VIII. p. 44. and *l.*7.44, *per annum,* according to the calculation in note F. at the end of the fecond volume of Obfervations on Reverfionary Payments, &c.

Thefe determinations exceed the income actually provided. But the exceffes are by no means confiderable enough to afford any certain reafon for concluding, that the fund of this eftablifhment will prove infufficient. I was, however, once led to entertain fome doubts on this fubject. And, in thofe doubts I thought myfelf confirmed by ob-

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ferving, that in the calculations (c) made at the commencement of the fcheme, the number 333 was ftated, as the maximum of widows living at one time, likely to come upon it, or to be derived from 20 widows left annually(d); and alfo, that 40 years was ftated as the number of years neceffary to bring on this maximum; whereas I was fatisfied, that the maximum of widows would not prove much lefs than 400 (e); nor the number of years neceffary to bring it on, lefs than 60. But I have lately received fuch information (f) as has convinced me that my doubts have been in a great meafure groundlefs. I have learned, in particular, that there have been feveral calculations fubfequent to thofe I had feen; and that this eftablifhment has enjoyed advantages and provifions for its fupport which I was unacquainted with, and which give reafon for expecting that it will, indeed, be able to bear the expence of 400 annuitants, fhould fo many

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(c) See Table III. in a book printed at Edinburgh, in 1748, entitled, Calculations, with the Principles and Data, on which they are inftituted, relative to a late Act of Parliament, entitled, "An Act for raifing and eftablifhing a Fund, for a provifion for the Widows and Children of the Minifters of the Church, and of the Heads, Principals, and Mafters of the Univerfities of Scotland; fhewing the Rife and Progrefs of the Fund."

(d) See note (k) page 36.

(e) The number of widows left annually multiplied by the expectation of widowhood, gives the maximum of widows.

(f) I owe this information to the kind and very obliging candour of the reverend and ingenious Dr. *Webfter*, of Edinburgh.

ny come upon it. I fhould only tire moſt of my readers were I to enter into an account of theſe advantages and proviſions. It will be of more importance to take this opportunity to obſerve, that the probabilities of life from which the determinations I have mentioned are derived, though much lower than the probabilities of life among the miniſters and their wives in Scotland, are yet ſuch as give the values of reverſions depending on ſurvivorſhips among them too high.

In order to underſtand this, it muſt be conſidered, that the difference between the probabilities of life, in different ſituations, takes place much more in the firſt and middle than in the laſt ſtages of life; and that the effect of this muſt be to increaſe the duration of *joint lives*, and at the ſame time to leſſen the duration of *ſurvivorſhip* in thoſe ſituations which are moſt favourable to health. Or in other words, to render the duration of *marriage* in ſuch ſituations, greater than it would otherwiſe be in proportion to the duration of *widowhood*. For inſtance. Were the probabilities of life among the miniſters and their wives in *Scotland* the ſame that they are in Mr. *De Moivre's* hypotheſis, or in the *Breſlaw* and *Northampton* Tables of Obſervation, the duration of marriages among them taken one with another, could not be more than 19 years. The duration of widowhood would be 22 years, and the maximum of widows living at one time derived from 667 marriages conſtantly kept up, would be conſiderably more than 400. Were the probabilities of life among them the

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fame that they are in *London*, the duration of marriage would be ftill lefs, and the duration of widowhood greater, and the maximum of widows derived from 667 marriages, could not be lefs than 500. But the fact is, that the duration of marriage among them is 22 years, nearly; and that of widowhood about 20 years and a half. And it appears alfo, from accounts taken annually, that the number of widows living at one time derived from the whole body of minifters and profeffors, does not exceed 400. It is, therefore certain, that a fmaller income muft be fufficient for the fupport of this fcheme than would be neceffary, according to the probabilities of life in the Tables juft mentioned. And upon the whole, after a careful review of all the circumftances of this eftablifhment in its prefent ftate, I am well fatisfied that the fuccefs with which it has been hitherto attended, is likely to continue; and that it will, indeed, prove a permanent foundation of that affiftance to the widow and fatherlefs which is intended by it. Caution, however, and vigilance will for fome time be neceffary. Many more years muft pafs before it can receive a decifive confirmation from experience. Events have hitherto favoured it more than could have been reafonably expected. They may perhaps hereafter try it; and deviations from probability may arife, which cannot now be forefeen.—But I ought to afk pardon for making thefe remarks. The venerable minifters and profeffors concerned, will, I hope excufe me. They are eminently diftinguifhed by their abilities and knowledge; and can have little need of any information which I am able to give them.

Vol. I. page 115. Dr. *Price* gives this further account of the Establishment among the Ministers and Professors in Scotland.

The establishment among the Ministers and Professors in Scotland, has prospered to a degree which gives reason to believe that it cannot fail to answer the hopes of the venerable body interested in it. This has been owing chiefly to the great ability and faithful zeal of the Rev. Dr. *Webster*, its founder and conductor. To the account already given of it, I will take this opportunity to add the following particulars.

Dr. *Webster* having when the plan was first formed, 38 years ago, no certain *data* to go upon assumed fifty-two as the medium age, at which the widows of ministers would commence annuitants. By calculating on this supposition, and taking the chances of life as they are in Dr. *Halley*'s Table, he found that the number of annuitants on the scheme at Lady-day 1780, would be 310. The fact is, that they were then 304 : and, that consequently there was even in this way of calculating, a difference of six in favour of the funds which support the scheme. Since the establishment of the scheme it has been discovered, that the medium age just mentioned does not probably exceed 47. Dr. *Webster*, therefore, some years ago, in order to put the scheme to a severer trial, instituted a new calculation, on the supposition that the medium age is no more than 44, and found, that on this supposition the number of annuitants at Whitsuntide, 1780, would be 328. This has
made

made a still greater difference in favour of the establishment, and gives a very encouraging prospect of its stability; a sufficient income having been in reality provided for bearing the expences, had the annuitants increased as in this last calculation. Had 52 been the mean age of the widows when they commence such, the *maximum* of widows living at one time derived from 20 left annually, would be 334, according to Dr. *Halley's* Table; but supposing it no more than 44, this *maximum* would exceed 400; and the enquiries which have been made, give reason to expect that it will not fall much short of this number. Dr. *Webster*, therefore, has, in his last calculations, reckoned upon the increase of the annuitants to this number, and for this reason, and to secure more certainly the establishment, a new Act of Parliament was procured in 1779, by which among other new provisions, it was ordered, that the increase of the capital, (then amounting to 75,088*l*(*g*). should not be discontinued till it rose to 100,000*l*. This capital joined to the annual contributions, will probably be an ample support to the establishment, should the number of annuitants (which will go on to encrease for near forty years more) become at last 400. Circumspection and caution, however, continue to be necessary, because still unfavourable

events

(*g*) *At the commencement of this establishment a calculation was made to determine what what would be the probable amount of the fund in each year, till the proposed capital should be compleated. It was calculated that in 1758, it would be 47401l. it actually was 47313l. It was calculated that in 1779, it would be 72508l. it actually was 75088l.*

events may arife, which no human wifdom can forefee.

Having beftowed a good deal of attention on this inftitution, I cannot take leave of it without congratulating Dr. *Webfter* on his happinefs, by being the founder of this fcheme, and by the care with which he has watched its progrefs and conducted it to its prefent ftate of maturity, he has entitled himfelf to the bleffings of many indigent widows and orphans, and made it impoffible that he fhould be ever remembered in the Church of *Scotland* without gratitude and refpect.

TABLES

T A B L E S,

Shewing the prefent Values of an annuity of 1*l.*
on a fingle Life, according to Mr. DE MOIVRE's
hypothefis ; and, therefore, nearly according to
the probabilities of life at BRESLAW, NORWICH,
and NORTHAMPTON.

T A B L E   I. (a)

Age.	3 per Cent.	3½ per Cent.	4 per Cent	4½ per Cent.	5 per Cent.	6 per Cent.
8	19,736	18,160	16,791	15,595	14,544	12,790
9	19,868	18,269	16,882	15,672	14,607	12,839
10	19,868	18,269	16,882	15,672	14,607	12,839
11	19,736	18,160	16,791	15,595	14,544	12,790
12	19,604	18,049	16,698	15,517	14,480	12,741
13	19,469	17,937	16,604	15,437	14.412	12,691
14	19,331	17,823	16,508	15,356	14,342	12,639
15	19,192	17,707	16,410	15,273	14,271	12,586
16	19,050	17,588	16,311	15,189	14,197	12,532
17	18,905	17,467	16,209	15,102	14,123	12,476
18	18,759	17,344	16,105	15,015	14,047	12,419
19	18,610	17,220	15,999	14,923	13,970	12,361
20	18,458	17,093	15,891	14,831	13,891	12,301
21	18,305	16,963	15,781	14,737	13,810	12,239
22	18,148	16,830	15,669	14,641	13,727	12,177
23	17,990	16,696	15,554	14,543	13,642	12,112
24	17,827	16,559	15,437	14,442	13,555	12,045
25	17,664	16,419	15,318	14,340	13,466	11,978
26	17,497	16,277	15,197	14,235	13,375	11,908
27	17,327	16,133	15,073	14,128	13,282	11,837
28	17.154	15,985	14,946	14,018	13,186	11,763
29	16,979	15,835	14,816	13,905	13,088	11,688
30	16,800	15,682	14,684	13,791	12,988	11,610
31	16,620	15,526	14,549	13,673	12,855	11,530
32	16,436	15,367	14,411	13,553	12,780	11,449
33	16,248	15,204	14,270	13,430	12,673	11,365

(a) This Table is the fame with Mr. *De Moivre's* Table of the values of fingle lives, publifhed in his "Treatife on Life Annuities," and carried as far as the age of 79 to 3 places of decimals by Mr. *Dodfon*, in his *Mathematical Repofitory*, vol. 2. page 169.

Age.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.
34	16,057	15,039	14,126	13,304	12,562	11,278
35	15,864	14,871	13,979	13,175	12,449	11,189
36	15,666	14,699	13,829	13,044	12,333	11,098
37	15,465	14,524	13,676	12,909	12,214	11,003
38	15,260	14,345	13,519	12,771	12,091	10,907
39	15,053	14,163	13,359	12,630	11,966	10,807
40	14,842	13,978	13,196	12,485	11,837	10,704
41	14,626	13,789	13,028	12,337	11,705	10,599
42	14,407	13,596	12,858	12,185	11,570	10,490
43	14,185	13,399	12,683	12,029	11,431	10,378
44	13,958	13,199	12,504	11,870	11,288	10,263
45	13,728	12,993	12,322	11,707	11,142	10,144
46	13,493	12,784	12,135	11,540	10,992	10,621
47	13,254	12,571	11,944	11,368	10,837	9,895
48	13,012	12,354	11,748	11,192	10,679	9,765
49	12,764	12,131	11,548	11,012	10,515	9,630
50	12,511	11,904	11,344	10,827	10,348	9,492
51	12,255	11,673	11,135	10,638	10,176	9,349
52	11,994	11,437	10,921	10,443	9,999	9,201
53	11,729	11,195	10,702	10,243	9,817	9,049
54	11,457	10,950	10,478	10,039	9,630	8,891
55	11,183	10,698	10,248	9,829	9,437	8,729
56	10,902	10,443	10,014	9,614	9,239	8,561
57	10,616	10,181	9,773	9,393	9,036	8,387
58	10,325	9,913	9,527	9,166	8,826	8,208
59	10,029	9,640	9,275	8,933	8,611	8,023
60	9,727	9,361	9,017	8,694	8,389	7,831
61	9,419	9,076	8,753	8,449	8,161	7,633
62	9,107	8,786	8,482	8,197	7,926	7,428
63	8,787	8,488	8,205	7,938	7,684	7,216
64	8,462	8,185	7,921	7,672	7,435	6,997
65	8,132	7,875	7,631	7,399	7,179	6,770
66	7,794	7,558	7,333	7,119	6,915	6,535
67	7,450	7,234	7,027	6,831	6,643	6,292
68	7,099	6,902	6,714	6,534	6,362	6,040
69	6,743	6,565	6,394	6,230	6,073	5,779
70	6,378	6,219	6,065	5,918	5,775	5,508

Age.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent,
71	6,008	5,865	5,728	5,596	5,468	5,228
72	5,631	5,505	5,383	5,265	5,152	4,937
73	5,246	5,136	5,029	4,926	4,826	4,636
74	4,854	4,759	4,666	4,576	4,489	4,324
75	4,453	4,373	4,293	4,217	4,143	4,000
76	4,046	3,978	3,912	3,847	3,784	3,664
77	3,632	3,575	3,520	3,467	3,415	3,315
78	3,207	3,163	3,111	3,076	3,034	2,953
79	2,776	2,741	2,707	2,673	2,641	2,578
80	2,334	2,309	2,824	2,259	2,235	2,188
81	1,886	1,867	1,850	1,832	1,816	1,783
82	1,429	1,411	1,406	1,394	1,384	1,362
83	0,961	0,955	0,950	0,943	0,937	0,925
84	0,484	0,483	0,481	0,479	0,476	0,472
85	0,000	0,000	0,000	0,000	0,000	0,000

TABLE II.

Shewing the value of an Annuity on the joint continuance of two lives, according to Mr. *De Moivre's Hypothesis*; and, therefore, nearly according to the probabilities of life at Breslaw, Norwich, and Northampton, p. 2, 3.

Age of the youngest	Age of the eldest	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
	10	15.206	13.342	11.855
	15	14.878	13.093	11.661
	20	14.503	12.808	11.430
	25	14.074	12.480	11.182
10	30	13.585	12.102	10.884
	35	13.025	11.665	10.537
	40	12.381	11.156	10.128
	45	11.644	10.564	9.646
	50	10.796	9.871	9.074
	55	9.822	9.059	8.391
	60	8.704	8.105	7.572
	65	7.417	6.980	6.585
	70	5.936	4.652	5.391
	15	14.574	12.860	11.478
	20	14.225	12.593	11.266
	25	13.882	12.281	11.022
	30	13.359	11.921	10.736
	35	12.824	11.501	10.402
15	40	12.207	11.013	10.008
	45	11.496	10.440	9.541
	50	10.675	9.767	8.985
	55	9.727	8.975	8.318
	60	8.632	8.041	7.515
	65	7.377	6.934	6.544
	70	5.932	5.623	5.364

Age of the youngest	Age of the eldest	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
	20	13.904	12.341	11.067
	25	13.531	12.051	10.840
	30	13.098	11.711	10.565
	35	12.594	11.314	10.278
	40	12.008	10.847	9.870
20	45	11.325	10.297	9.420
	50	10.536	9.648	8.880
	55	9.617	8.879	8.233
	60	8.549	7.967	7.448
	65	7.308	6.882	6.495
	70	5.868	5.590	5.333
	25	13.192	11.786	10.621
	30	12.794	11.468	10.367
	35	12.333	11.093	10.067
	40	11.776	10.655	9.708
25	45	11.130	10.131	9.278
	50	10.374	9.509	8.761
	55	9.488	8.766	8.134
	60	8.452	7.880	7.371
	65	7.241	6.826	6.440
	70	5.826	5.551	5.294
	30	12.434	11.182	10.133
	35	12.010	10.838	9.854
	40	11.502	10.428	9.515
	45	10.898	9.935	9.112
30	50	10.183	9.345	8.620
	55	9.338	8.634	8.018
	60	8.338	7.779	7.280
	65	7.161	6.748	6.373
	70	5.777	5.505	6.254

Age of the youngest	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
	35	11.632	10.530	9.600
	40	11.175	10.157	9.291
	45	10.622	9.702	8.913
35	50	9.955	9.149	8.450
	55	9.156	8.476	7.879
	60	8.202	7.658	7.172
	65	7.066	6.662	6.294
	70	5.718	5.450	5.203
	40	10.777	9.826	9.014
	45	10.283	9.418	8.671
40	50	9.677	8.911	8.244
	55	8.936	8.283	7.710
	60	8.038	7.510	7.039
	65	6.951	6.556	6.198
	70	5.646	5.383	5.141
	45	9.863	9.063	1.370
	50	9.331	8.619	7.987
45	55	8.662	8.044	7.500
	60	7.831	7.332	6.875
	65	6.807	6.425	6.080
	70	5.556	5.300	5.063
	50	8.892	8.235	7.660
	55	8.312	7.738	7.230
50	60	7.568	7.091	6.664
	65	6.623	6.258	5.926
	70	5.442	5.193	4.964
	55	7.849	7.332	6.873
	60	7.220	6.781	6.386
55	65	6.379	6.036	5.724
	70	5.291	5.053	3.833

TABLE II. Continued.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
60	60	6.737	6.351	6.001
	65	6.043	5.730	5.444
	70	5.081	4.858	4.653
65	65	5.547	5.277	5.031
	70	4.773	4.571	4,385
70	70	4.270	4.104	3.952

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